SM 122 Quiz #4A (26 Jan 04: TSM) Spring 2004 Last Name: SOLUTION

Time: 4 minutes: Closed book, closed notes, no calculator allowed

1. Let $f(x)$ be a continuous function on the interval $[a, b]$.
   Complete the statements of the two parts of the Fundamental Theorem of Calculus.

   (I) If $g(x) = \int_a^x f(t)\,dt$, then $g'(x) = f(x)$

   (II) If $F(x)$ is an antiderivative of $f(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

2. What is the value of $G'(4)$ if

$$G(x) = \int_1^{x/2} \frac{1}{t^2+1}\,dt \quad ?$$

CIRCLE ONE AND PUT YOUR WORK BELOW

(a) 1/37
(b) 1/20
(c) 1/17
(d) $\boxed{1/10}$
(e) 1/5

By the Fundamental Theorem of Calculus and the Chain Rule

$$G'(x) = \frac{1}{\left(\frac{x}{2}\right)^2+1} \cdot \frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{\left(\frac{x}{2}\right)^2+1} \cdot \frac{1}{2}$$

So

$$G'(4) = \frac{1}{2^2+1} \cdot \frac{1}{2} = \frac{1}{10}$$